7.6 Properties of Systems of Linear Equations

Quick review before we begin:

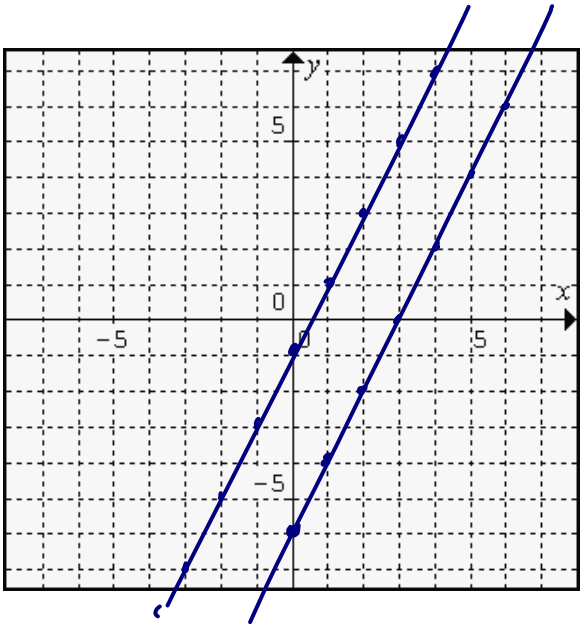
1. Name the three methods to solve a system of equations:
   a) GRAPHING
   b) ELIMINATION
   c) SUBSTITUTION

Example: Graph the following systems to find a solution:

Ex 1: $2x - y = 6$
$6x - 3y = 3$

$-y = -2x + 6$
$y = 2x - 6$

$-3y = -6x + 3$
$y = 2x - 1$



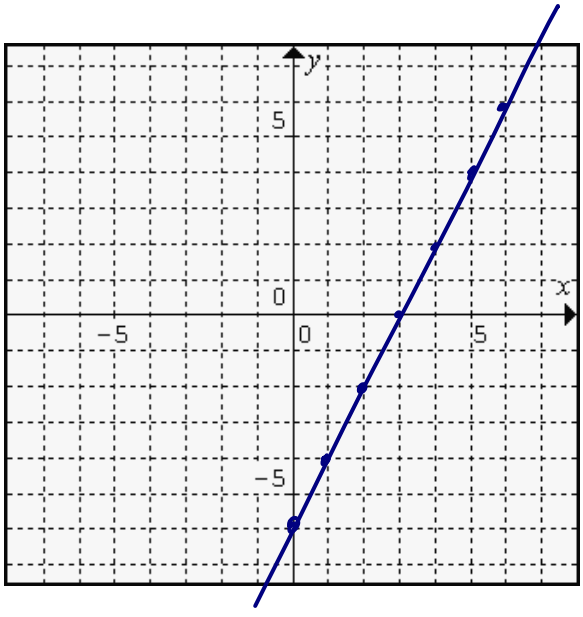
# of solutions: 0

PARALLEL LINES.
SAME SLOPE!

Ex 2: $2x - y = 6$
$6x - 3y = 18$

$y = 2x - 6$

$-3y = -6x + 18$
$y = 2x - 6$



INFINITELY MANY (∞)

SAME LINE.

When we solve a linear system, there are three possible outcomes:

1) Intersecting lines occurs when we have different slopes.

   Here there are 1 solution(s).

2) Parallel lines occur when we have SAME slopes and

   Different y-intercepts. Here there are 0

   solution(s).

3) Coincident lines occur when we have SAME

   slopes and SAME y-intercepts. Here there are

   ∞ solution(s).

Ex: Consider the equation 3x + y = 3. Write a second equation to form a linear system with:

a) infinitely many solutions
b) no solutions
c) one solution

$3x+y=3$

SLOPE INTERCEPT: $y=-3x+3$

(a) ∞ MANY.

$2y=-6x+6$

$3y=-9x+9$

$-y=3x-3$

(b) NO SOLUTIONS

$y=-3x+4$

$y=-3x+122$

(c) ONE SOL'N

$y=2x+1$

$y=4x+5$

Practice:

1. Determine whether each linear system has infinitely many solutions, no solutions or one solution.

a. $x + 2y = 6$
$x + 2y = 2$

b. $3x + 5y = 9$
$6x + 10y = 18$

c. $2x - 5y = 30$
$4x - 10y = 15$

d. $3x + 2y = 12$
$6x + 4y = 24$

2. Six equations are listed. Using each equation only once, write linear systems that have:

$4x + 2y = 20$
$x - 3y = 12$
$5x - 15y = -60$
$2x + y = 10$
$6x + 3y = 5$
$2x - 6y = 24$

a. No solution

b. One solution

c. Infinitely many solutions

3. Use the equation $3x - 4y = 8$. Write a second equation to form a linear system that has:

a. No solution

b. One solution

c. Infinitely many solutions

4. Two linear equations have the same slope. What other information do you need to know to determine if the equations have no solution or infinitely many solutions?

5. For which of these is (2, –3) a solution?

a. $\begin{aligned} x - y &= 5 \\ 3x + 4y &= -6 \end{aligned}$ b. $\begin{aligned} 2x + y &= 7 \\ x - 3y &= 10 \end{aligned}$ c. $\begin{aligned} 4x - y &= 11 \\ -12x + 3y &= -33 \end{aligned}$ d. $\begin{aligned} 5x - 3y &= 19 \\ -2x + 4y &= -16 \end{aligned}$

6. For Question 5, which system is (2, –3) the only solution?

Extra Practice: p. 448 # 6, 7, 11 – 16